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# Appendix H: Estimation of Usage Impacts In PRISM

## Quantifying the Benefits of Dynamic Pricing In the Mass Market

Prepared by:
**Sanem Sergici**

***The Brattle Group***

Prepared for:
**Edison Electric Institute**

**January 2008**

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Published by:
Edison Electric Institute
701 Pennsylvania Avenue, N.W.
Washington, D.C. 20004-2696
Phone: 202-508-5000
Web site: www.eei.org

# APPENDIX H: ESTIMATION OF USAGE IMPACTS IN PRISM

The PRISM model predicts the changes in electricity usage that are induced by time-varying rates.[1] One important feature of PRISM is its capability to model nonlinearities in the estimation of usage impacts when price changes extend from minimal to maximal. In PRISM, the usage impacts increase at a decreasing rate when prices are increased in a linear fashion. This nonlinear relationship is reflected in the shape of response curves that show the percentage impact on load due to price changes. In this appendix, we briefly describe the demand equations in PRISM and demonstrate through the use of demand curves, response curves, and own-price elasticities how PRISM handles nonlinearities in the estimation of usage impacts.

The PRISM model predicts the usage impacts by utilizing the parameter estimates of a constant elasticity of substitution (CES) demand system.[2] This demand system consists of two equations. The substitution equation predicts the ratio of peak to off-peak quantities as a function of the ratio of peak to off-peak prices and other factors. The daily energy usage equation predicts the daily electricity usage as a function of daily price and other factors. Once the demand system is estimated, resulting equations are solved to determine the changes in electricity usage associated with a time-varying rate. Derivations for these equations are provided at the end of this appendix.

Parameter estimates from the demand system may also be used to infer two practical measures of customer responsiveness to the time-varying rates: the elasticity of substitution and the daily price elasticity. The *elasticity of substitution* can be calculated from the substitution equation and indicates the percentage change in the peak to off-peak usage ratio due to 1 percent change in the peak to off-peak price ratio. The *daily price elasticity* can be calculated from the daily usage equation and shows the percentage change in daily usage associated with 1 percent change in daily electricity prices. Although informative about substitution and conservation patterns of customers, substitution and daily usage elasticities are less direct measures of price responsiveness. The two more common measures of price responsiveness are the own- and cross-price elasticities of demand. For small price changes, it is possible to infer own- and cross-price elasticities from substitution and daily price elasticities (these derivations are provided at the end of this appendix). However, when price changes are large; it is more accurate to derive these elasticities through impact simulation for different prices and applying elasticity formulas. For large price changes, the arc elasticity approach provides more accurate elasticity estimates compared to the point elasticity approach. The arc elasticity formula uses the average of the initial and final points as a base when calculating the percentage changes in quantity and price whereas the point elasticity formula uses the initial point as the base for the same calculation.

[1] PRISM has the capability to predict these changes for peak and off-peak hours for both critical and non-critical peak days. Moreover, PRISM allows predictions to vary by other exogenous factors such as the saturation of central air conditioning and variations in climate.

[2] For the description of the CES model, see Charles River Associates, "Impact Evaluation of the California Statewide Pricing Pilot," March 2005.

In our demonstrations, we adopt the arc elasticity approach to calculate the own-price elasticity of demand. Both formulas are provided below:

$$\text{Arc Elasticity} = \frac{Q_2 - Q_1}{(Q_1 + Q_2)/2} \Big/ \frac{P_2 - P_1}{(P_1 + P_2)/2}$$

$$\text{Point Elasticity} = \frac{Q_2 - Q_1}{Q_1} \Big/ \frac{P_2 - P_1}{P_1}$$

Below we summarize our simulation results by introducing demand curves, response curves, and own-price elasticities for each of the peak and off-peak time periods on critical pricing days for the average customer, for customers with central air conditioning (CAC), and for customers without CAC.

## Demand Curves for Peak and Off-Peak Periods on Critical Days

The demand curves in Figure H-1 represent how the peak demand varies with the peak period price on critical days; all other factors are held constant. At a price of $0.13 per kWh, the peak price under the non-time-varying rate, the starting values for peak period energy use are 1.22 kWh per hour for the average customer, 1.32 kWh per hour for CAC customers, and 0.85 kWh per hour for non-CAC customers. At a price of $1.30 per kWh, a hypothetical critical peak price, peak consumption falls to 1.03 kWh, 0.96 kWh, and 0.78 kWh for the average, CAC, and non-CAC customers, respectively. As shown in Figure H-1, the ownership of CAC affects the slope of the demand curves. The demand curve is flatter for CAC customers who are more price-responsive and therefore reduce their peak demand more in response to time-varying rates. In contrast, the demand curve for non-CAC customers is steeper, implying less price responsiveness. Figure H-2 shows off-peak period demand curves on critical days. The differences in the slopes still hold for different types of customers but are less prominent in the case of off-peak demand. The more vertical shape of these curves implies that off-peak demand is less sensitive to time-varying rates compared to peak demand.

Figure H-1. Residential Customer Peak Demand Curves on Critical Days

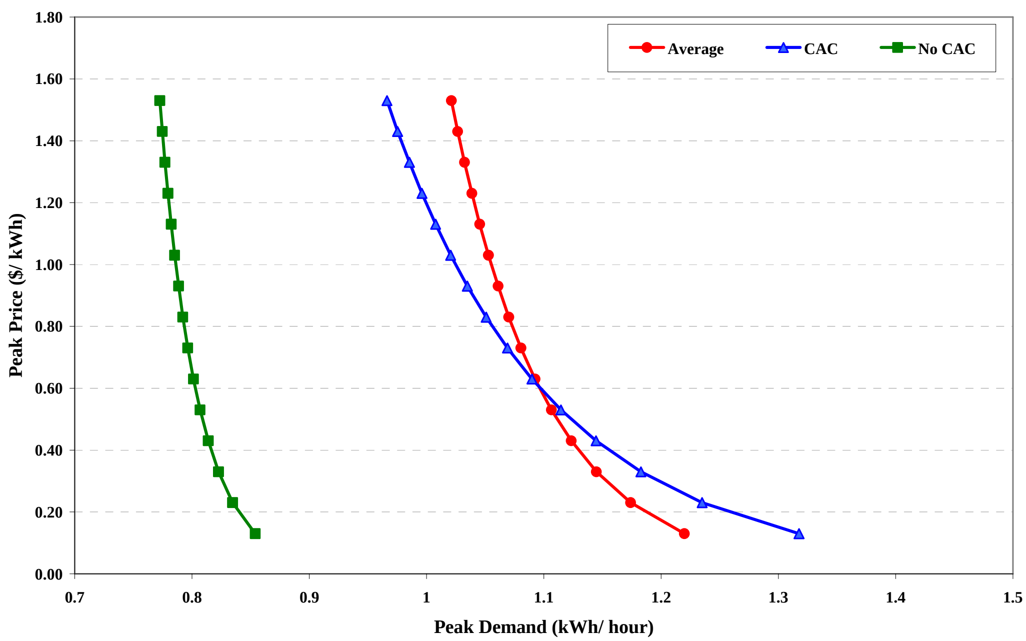
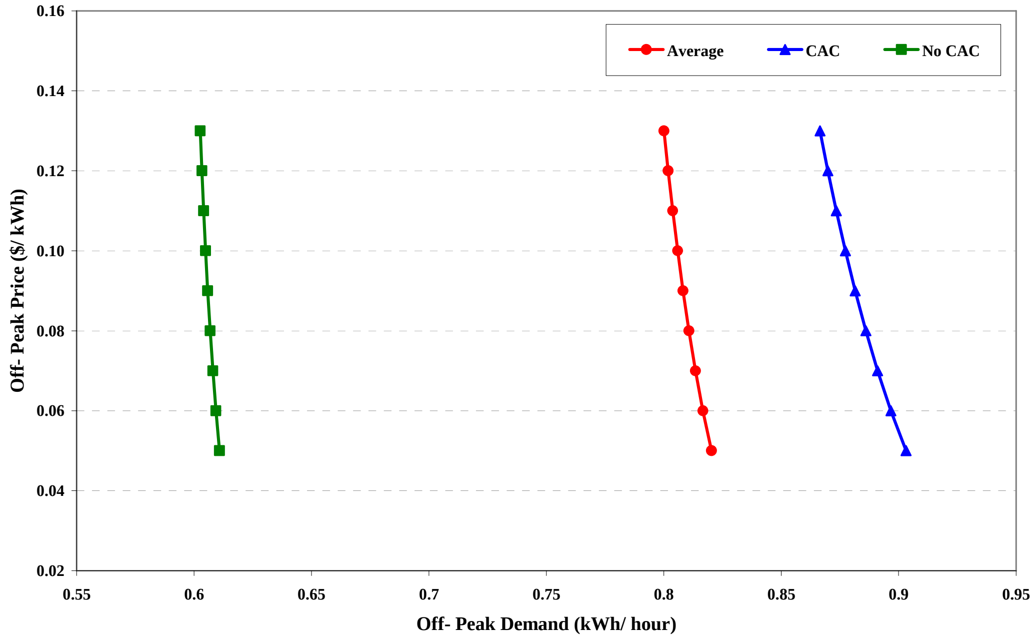


Figure H-2. Residential Customer Off-Peak Demand Curves on Critical Days



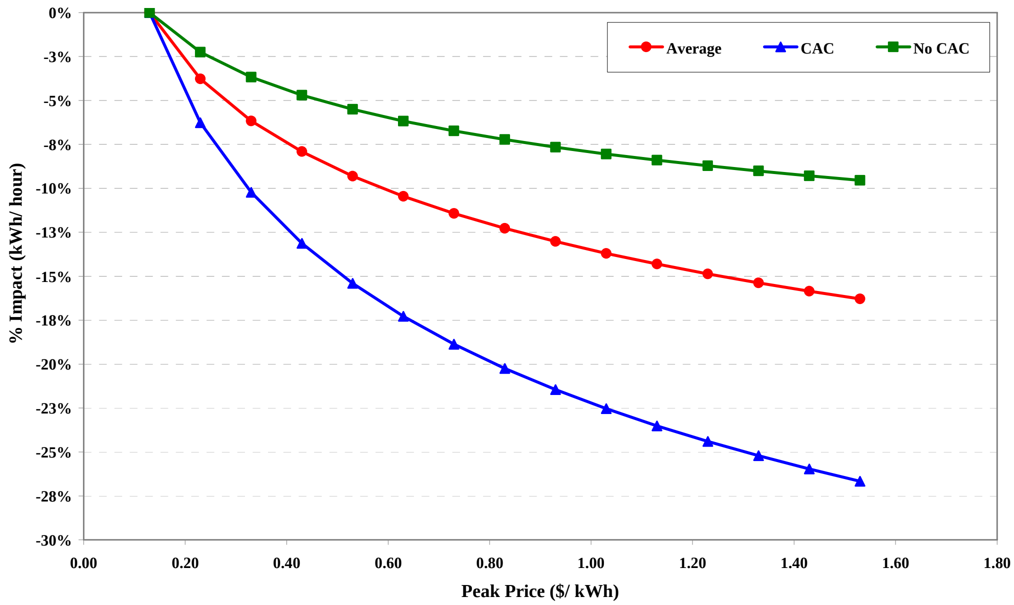

## Response Curves for Peak and Off-Peak Periods on Critical Days

The response curves in Figure H-3 demonstrate how the percentage impact on peak period energy usage varies with the peak-period price on critical days. These curves show that the percentage impact on the peak period energy usage increases as prices increase, but at a decreasing rate. This nonlinear relation between usage impacts and prices is reflected in the concave shape of the response curves.

To clarify how PRISM models the relationship between the prices and the percentage impact on load in a nonlinear fashion, consider the following example.

For the average customer, peak period energy usage decreases by 4 percent when the peak price increases from $0.13 per kWh to $0.23 per kWh. However, peak period energy usage decreases by only 8 percent when the peak price is increased from $0.13 per kWh to $0.43 per kWh. This example demonstrates that the load impact increases by onefold (rather than twofold) when the price increases by twofold.

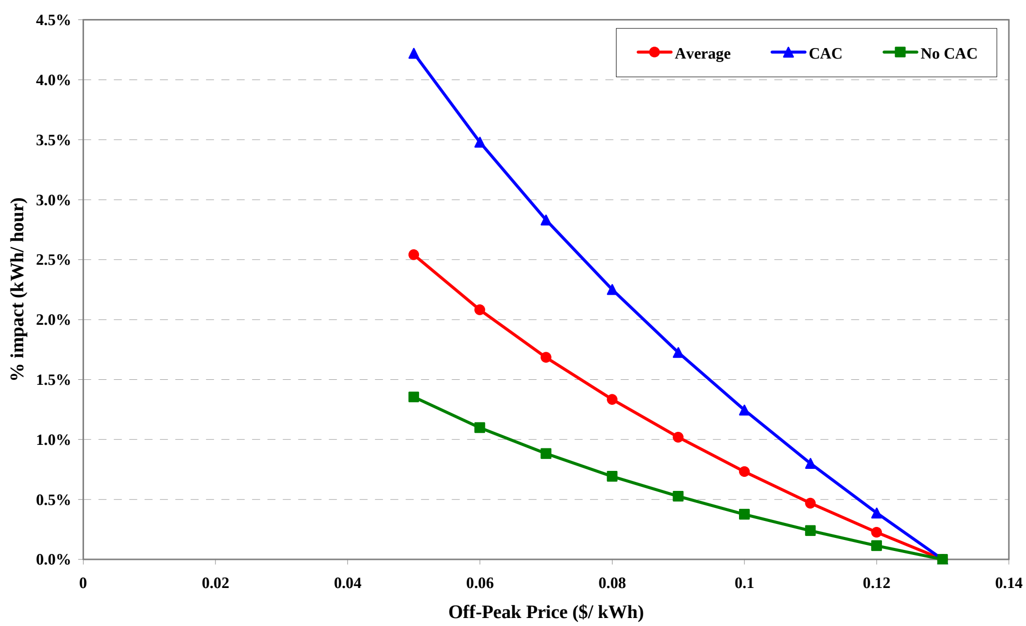
Figure H-3. Residential Customer Peak Response Curves on Critical Days




We can also observe the differences between customer types in their price responsiveness from these response curves. For a given price increase, non-CAC customers are the least responsive while CAC customers are the most responsive.

Figure H-4 shows the response curves for the off-peak period on critical days. Similar to the peak response curves, the off-peak response curves are also nonlinear. However, the flatter off-peak response curves in Figure H-4 demonstrate the limited price responsiveness during off-peak periods (compared to peak periods).

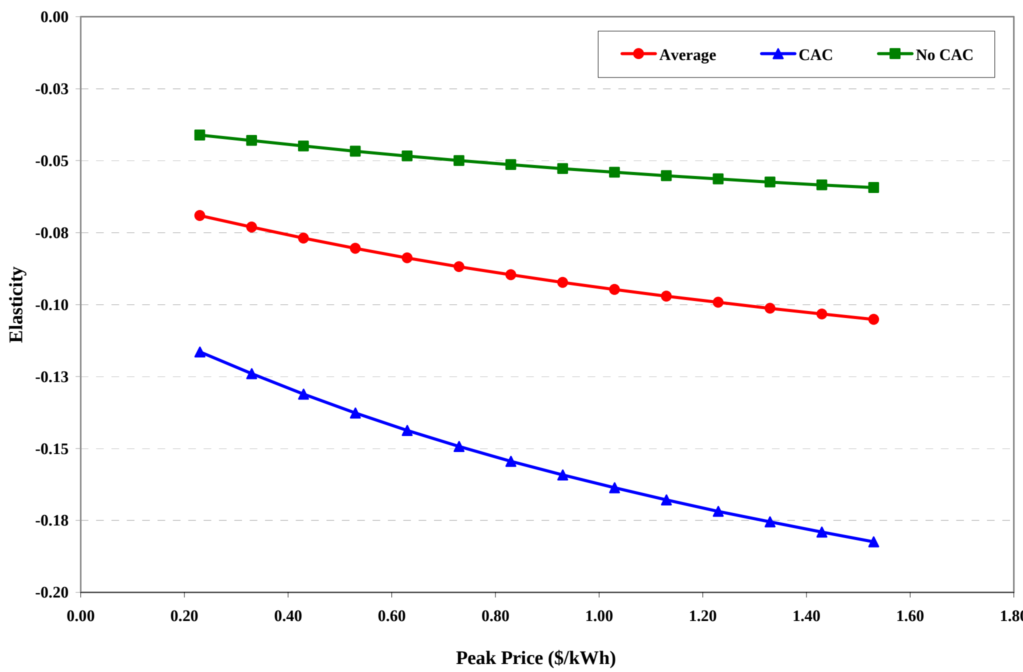
Figure H-4. Residential Customer Off-Peak Response Curves on Critical Days



## Own-price Elasticities on Critical Days

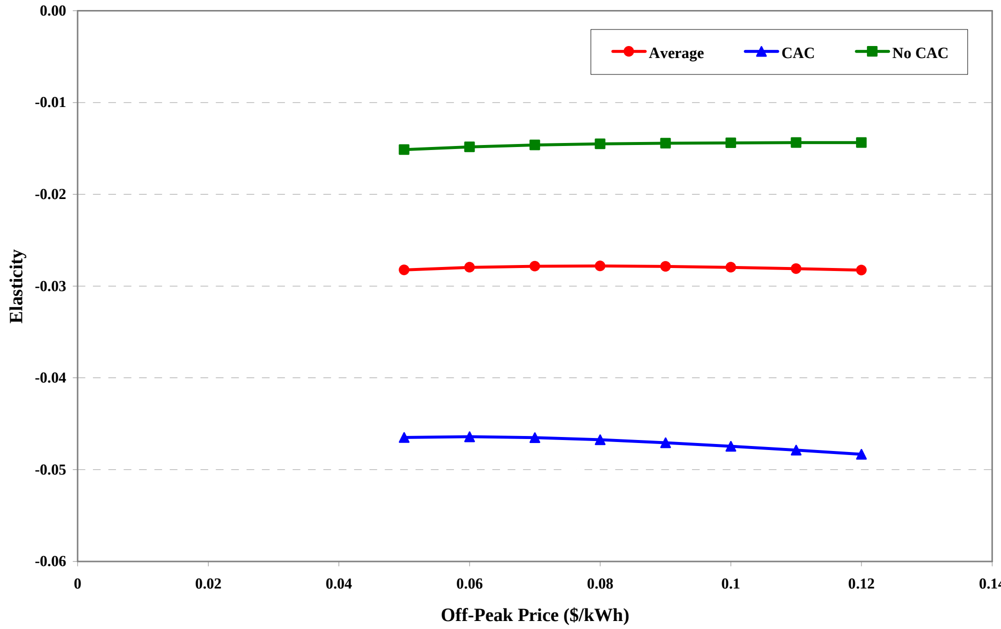
The own-price elasticity of demand is the most straightforward measure of price responsiveness. It shows the percentage change in demand due to 1 percent change in price. It is possible to derive an own-price elasticity of demand through simulation when there are large changes in price. We calculate the own-price elasticity of demand for peak and off-peak periods by simulating the changes in electricity usage for different price levels and then applying the arc price elasticity formula to the simulated impacts and prices. For the impact simulation, we start with a price of $0.13 per kWh and increase the price by 10-cent increments to a maximum of $1.53 per kWh. Figure H-5 shows the resulting own-price elasticities of peak demand by customer type. The elasticities range from -0.07 to -0.11 for average customers, -0.12 to -0.18 for CAC customers, and -0.04 to -0.06 for non-CAC customers. Consistent with the prior results, CAC customers have the most elastic peak demand while the non-CAC customers have the least elastic peak demand.

Figure H-5. Implicit Own Price Elasticity for Peak Demand on Critical Days




For the off-peak impact simulation, we start with $0.13 per kWh and decrease the price by one-cent increments to a maximum of $0.05 per kWh. Figure H-6 shows the resulting own-price elasticities of off-peak demand by customer. The own-price elasticities are negative but relatively flat – approximately -0.03 for average customers, -0.05 for CAC customers, and -0.015 for non-CAC customers.

Figure H-6. Implicit Own-price Elasticity for Off-peak Demand on Critical Days




## Derivation of PRISM Demand Equations

This section presents the derivation of the equations in the PRISM model to predict the changes in electricity usage as a result of time-varying prices.[3] Derivations are provided for a general, three-period TOU pricing scheme. The two-period case where there are only peak and off-peak periods is a special case of this general model.

When the TOU pricing scheme includes only two periods (peak and off-peak), only the elasticity of substitution between peak and off-peak usage is estimated. Thus, when the following equations are applied, we assume that $b_{12} = b_{32}$.

**In the reference case with no time varying prices, the following relationships hold:**

$$\ln\left(\frac{Q_1}{Q_2}\right) = a_{12} + b_{12}\ln\left(\frac{P_1}{P_2}\right) \qquad (0.1)$$

$$\ln\left(\frac{Q_3}{Q_2}\right) = a_{32} + b_{32}\ln\left(\frac{P_3}{P_2}\right) \qquad (0.2)$$

where $Q_i$ = Energy Usage in Period $i$, and $P_i$ = Price per Unit of Energy in Period $i$.

Also,

$$\bar{Q} = Q_1 + Q_2 + Q_3 \qquad (0.3)$$

[3] This section heavily draws on Appendix 8 of Charles River Associates, "Impact Evaluation of the California Statewide Pricing Pilot," March 2005.

**When time-varying prices are introduced (denoted by primes), the following relationships hold:**

$$\ln\left(\frac{Q_1^{'}}{Q_2^{'}}\right) = a_{12} + b_{12}\ln\left(\frac{P_1^{'}}{P_2^{'}}\right) \tag{0.4}$$

$$\ln\left(\frac{Q_3^{'}}{Q_2^{'}}\right) = a_{32} + b_{32}\ln\left(\frac{P_3^{'}}{P_2^{'}}\right) \tag{0.5}$$

and,

$$\overline{Q}^{'} = Q_1^{'} + Q_2^{'} + Q_3^{'} \tag{0.6}$$

**We start the derivations by subtracting (1.1) from (1.3):**

$$\left(\ln\left(\frac{Q_1^{'}}{Q_2^{'}}\right) = a_{12} + b_{12}\ln\left(\frac{P_1^{'}}{P_2^{'}}\right)\right) - \left(\ln\left(\frac{Q_1}{Q_2}\right) = a_{12} + b_{12}\ln\left(\frac{P_1}{P_2}\right)\right) \tag{0.7}$$

$$\ln\left(\frac{Q_1^{'}}{Q_2^{'}}\right) = \ln\left(\frac{Q_1}{Q_2}\right) + b_{12}\left(\ln\left(\frac{P_1^{'}}{P_2^{'}}\right) - \ln\left(\frac{P_1}{P_2}\right)\right) \tag{0.8}$$

**We set the right hand side of (1.8) to** $A_{12}$ :

$$A_{12} = \ln\left(\frac{Q_1}{Q_2}\right) + b_{12}\left(\ln\left(\frac{P_1^{'}}{P_2^{'}}\right) - \ln\left(\frac{P_1}{P_2}\right)\right) \tag{0.7}$$

$$\ln\left(\frac{Q_1^{'}}{Q_2^{'}}\right) = A_{12} \quad \text{or} \quad \ln(Q_1^{'}) = A_{12} + \ln(Q_2^{'}) \tag{0.8}$$

**Now, we exponentiate (1.10) and arrive at:**

$$\exp\ln(Q_1^{'}) = \exp(A_{12} + \ln(Q_2^{'})) \tag{0.9}$$
$$\Rightarrow Q_1^{'} = e^{A_{12}} Q_2^{'}$$

**Through a similar process, we can arrive at:**

$$\ln\left(\frac{Q_3^{'}}{Q_2^{'}}\right) = A_{32} \qquad (0.10)$$

$$A_{32} = \ln\left(\frac{Q_3}{Q_2}\right) + b_{32}\left(\ln\left(\frac{P_3^{'}}{P_2^{'}}\right) - \ln\left(\frac{P_3}{P_2}\right)\right) \qquad (0.11)$$

$$\exp\ln(Q_3^{'}) = \exp(A_{32} + \ln(Q_2^{'})) \qquad (0.12)$$
$$\Rightarrow Q_3^{'} = e^{A_{32}} Q_2^{'}$$

**This leaves us with:**

$$Q_1^{'} = e^{A_{12}} Q_2^{'}$$

and

$$Q_3^{'} = e^{A_{32}} Q_2^{'}$$

**Then we insert both of these equations into (1.3):**

$$\overline{Q}^{'} = e^{A_{12}} Q_2^{'} + Q_2^{'} + Q_2^{'} e^{A_{32}} \qquad (0.13)$$

$$\overline{Q}^{'} = Q_2^{'}(1 + e^{A_{12}} + e^{A_{32}}) \qquad (0.14)$$

$$Q_2^{'} = \frac{\overline{Q}^{'}}{(1 + e^{A_{12}} + e^{A_{32}})} \qquad (0.15)$$

**We finally arrive at:**

$$Q_1^{'} = e^{A_{12}} Q_2^{'}$$
$$Q_2^{'} = \frac{\overline{Q}^{'}}{(1 + e^{A_{12}} + e^{A_{32}})} \qquad (0.16)$$
$$Q_3^{'} = e^{A_{32}} Q_2^{'}$$

The two-period case is a special case of this set of relationships where $A_{32} = 0$.

## Derivation of Own- and Cross-price Elasticities from PRISM Demand Equations

Point estimates of the own-price and cross-price elasticities of demand can be derived from the CES demand model.[4] The first equation of the demand model expresses the ratio of energy usage in each rate period as a function of the ratio of prices in each period, while the second equation expresses daily electricity consumption as a function of daily electricity prices. It is important to note that the equations presented in this section are based on energy usage for each rate period, rather than energy usage per hour.

**The first equation of the CES demand system:**

$$\ln\left(\frac{Q_p}{Q_{op}}\right) = a + b\ln\left(\frac{P_p}{P_{op}}\right) \quad (2.1)$$

where:

$Q_p =$ Peak period energy use

$Q_{op} =$ Off-peak period energy use

$P_p$ = Peak period energy price

$P_{op}$ = Off-peak period energy price

**If there are two usage periods, then the following identity holds:**

$$Q_d = Q_p + Q_{op} \quad (2.2)$$

where,

$Q_d$ =Daily energy use

**The second equation of the demand system:**

$$\ln Q_d = c + d\ln(P_d) \quad (2.3)$$

where:

$$P_d = w_p P_p + w_{op} P_{op} \quad (2.4)$$

$P_d$ = Average daily electricity price

$w_p$ = total peak period electricity use

$w_{op}$ =total off-peak period electricity use

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[4] This section heavily draws on Appendix 9 of Charles River Associates, “Impact Evaluation of the California Statewide Pricing Pilot,” March 2005.

**Then, we define the following budget shares:**

$$z_p = \left( \frac{w_p P_p}{w_p P_p + w_{op} P_{op}} \right) \tag{2.5}$$

$$z_{op} = \left( \frac{w_{op} P_{op}}{w_p P_p + w_{op} P_{op}} \right) \tag{2.6}$$

Using the relevant equations and applying the chain rule, we derive the following expressions for the own- and cross-price elasticities of demand:

Own-price elasticity in the peak period: $$\eta_p = \frac{\partial \ln Q_p}{\partial \ln P_p} = w_{op} b + d z_p \tag{2.7}$$

Own-price elasticity in the off-peak period: $$\eta_{op} = \frac{\partial \ln Q_{op}}{\partial \ln P_{op}} = w_p b + d z_{op} \tag{2.8}$$

Cross-price elasticity in the peak period: $$\eta_{p,op} = \frac{\partial \ln Q_p}{\partial \ln P_{op}} = -w_{op} b + d z_{op} \tag{2.9}$$

Cross-price elasticity in the off-peak period: $$\eta_{op,p} = \frac{\partial \ln Q_{op}}{\partial \ln P_{op}} = -w_p b + d z_p \tag{2.10}$$